미시세계와 거시세계
6a. Fourier Series
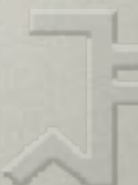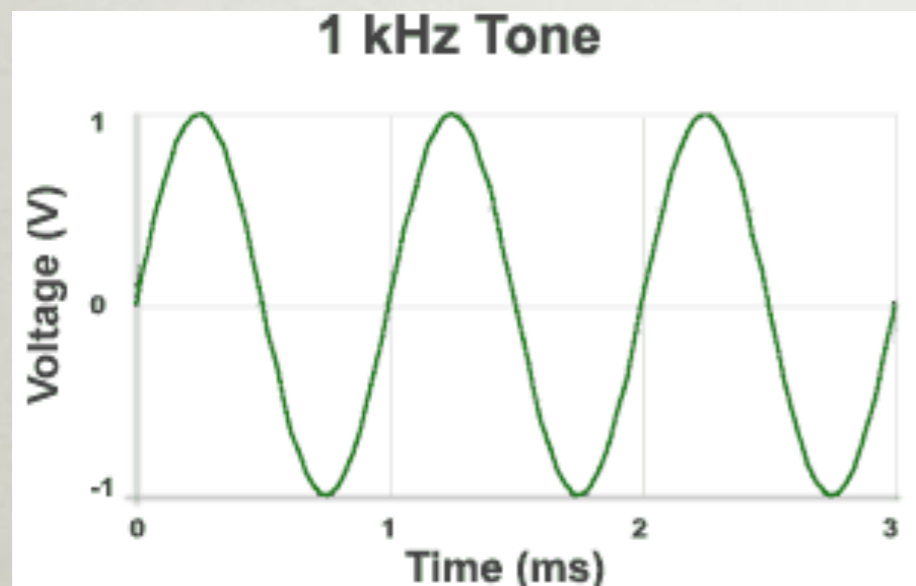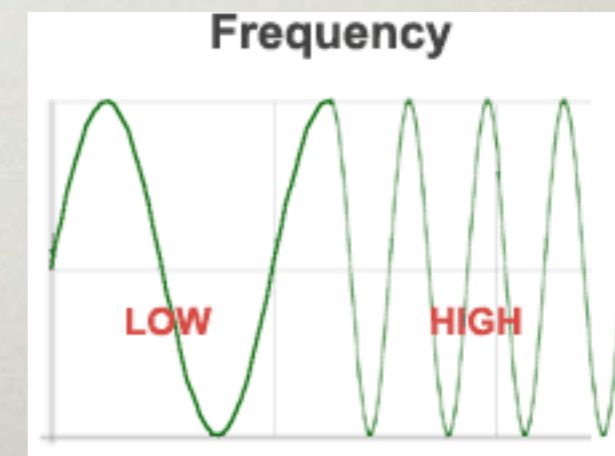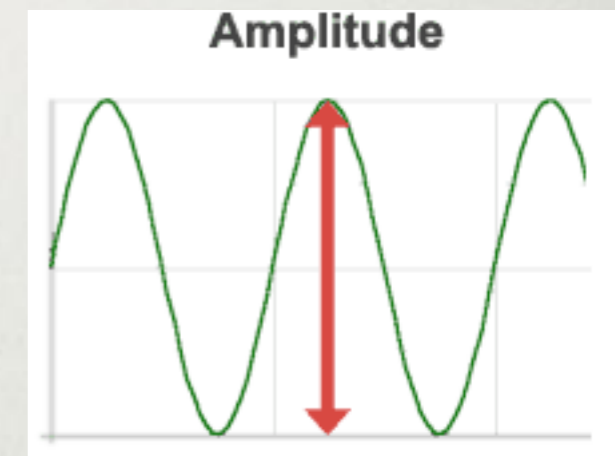
유재준

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2016/2학기

How sound wave propagates

Wavelength

Amplitude

1 kHz Tone

Frequency

LOW   HIGH
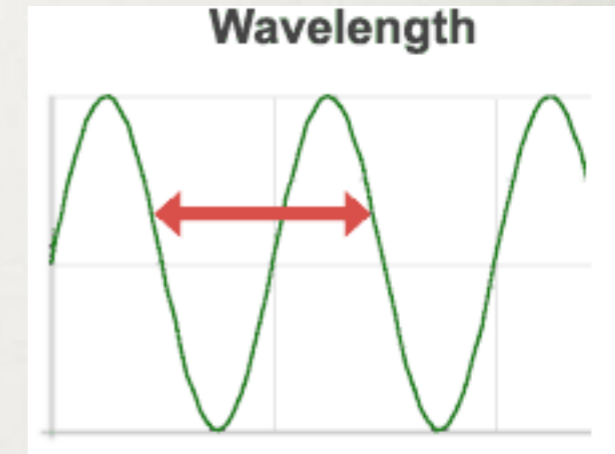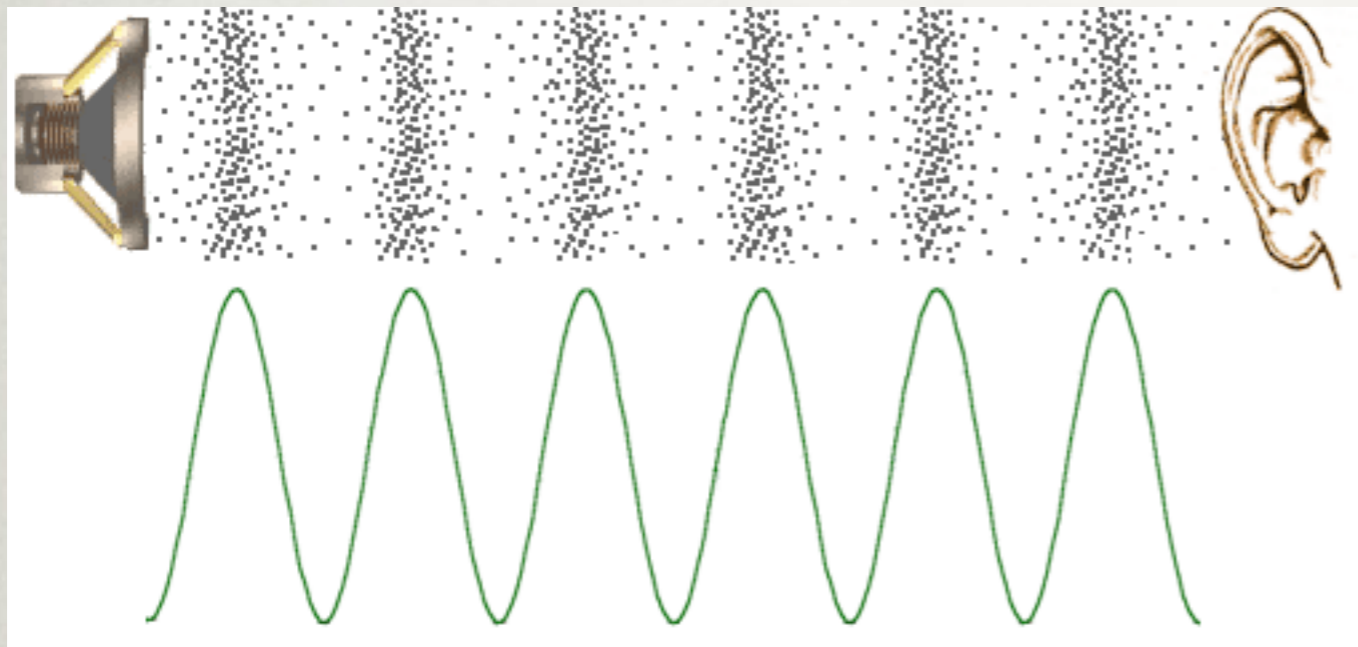
진동이란? What vibrates?
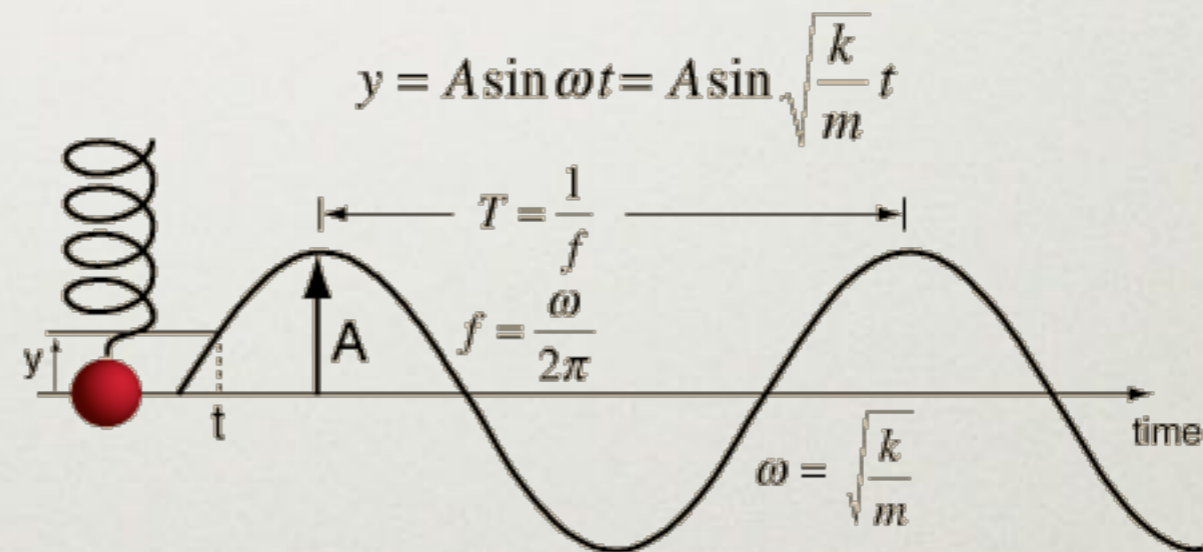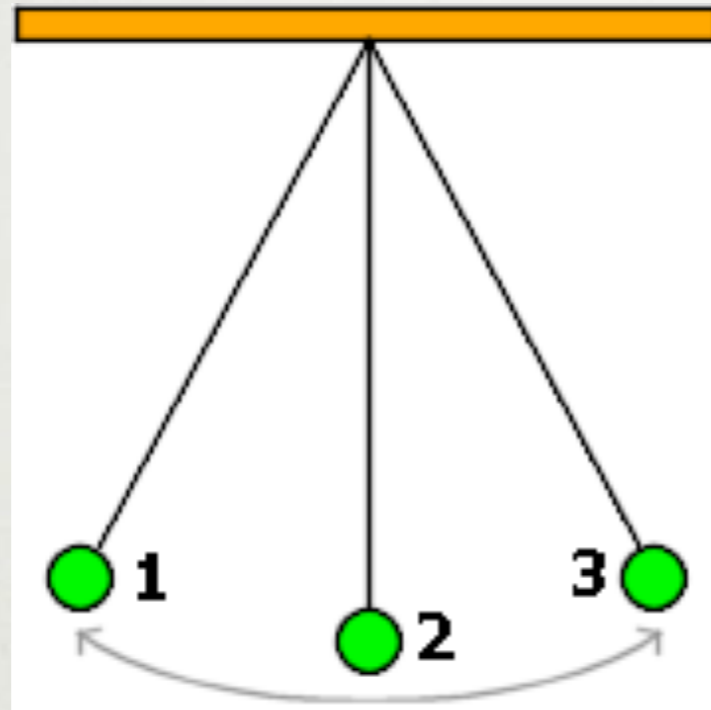
소리 스펙트럼
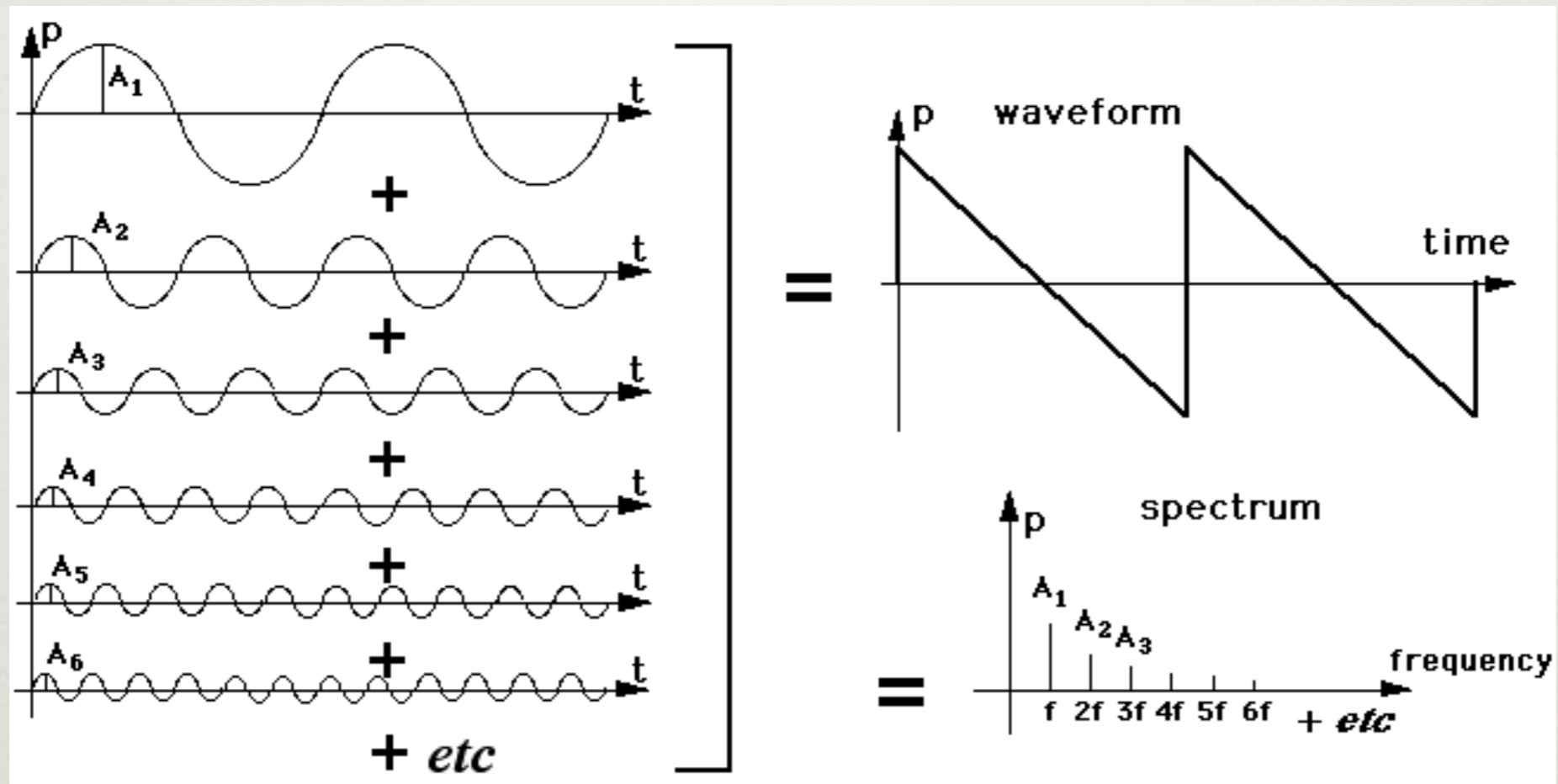
A sawtooth wave

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(nx) + b_n \sin(nx) \right]$$
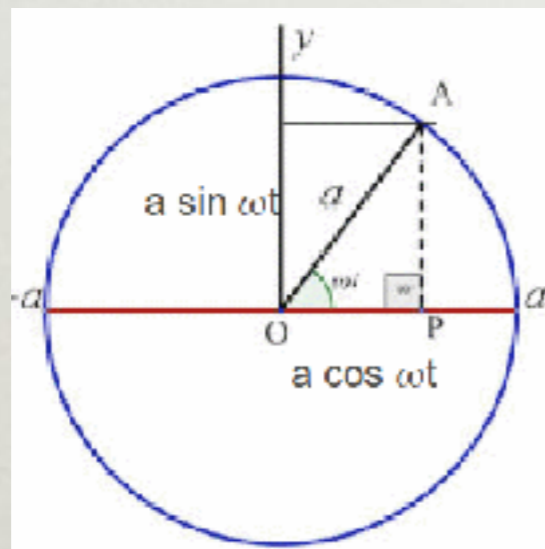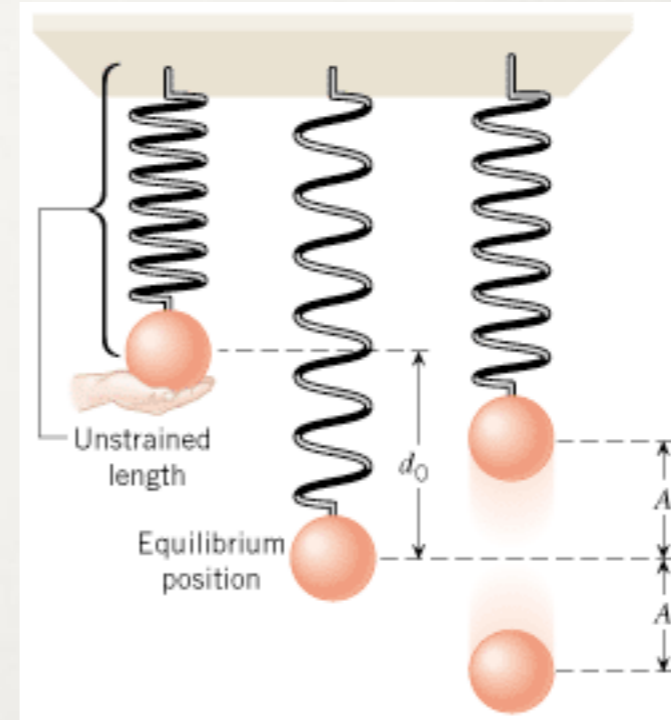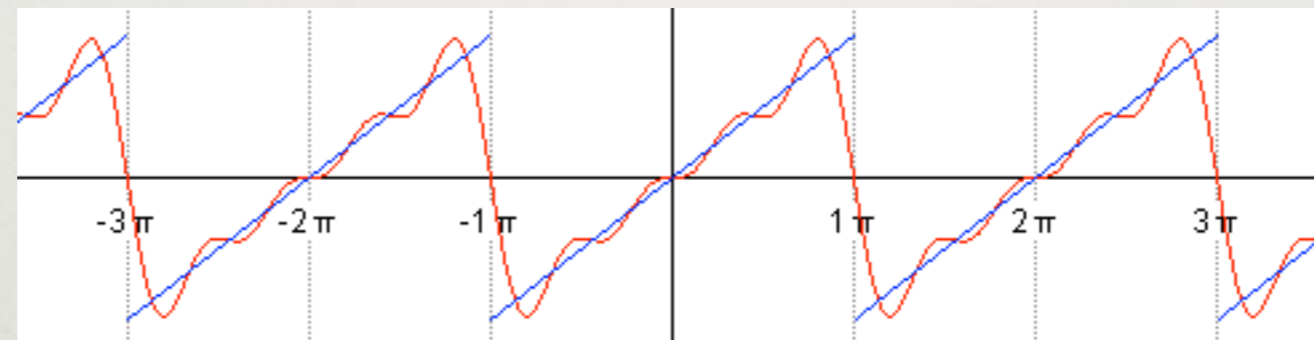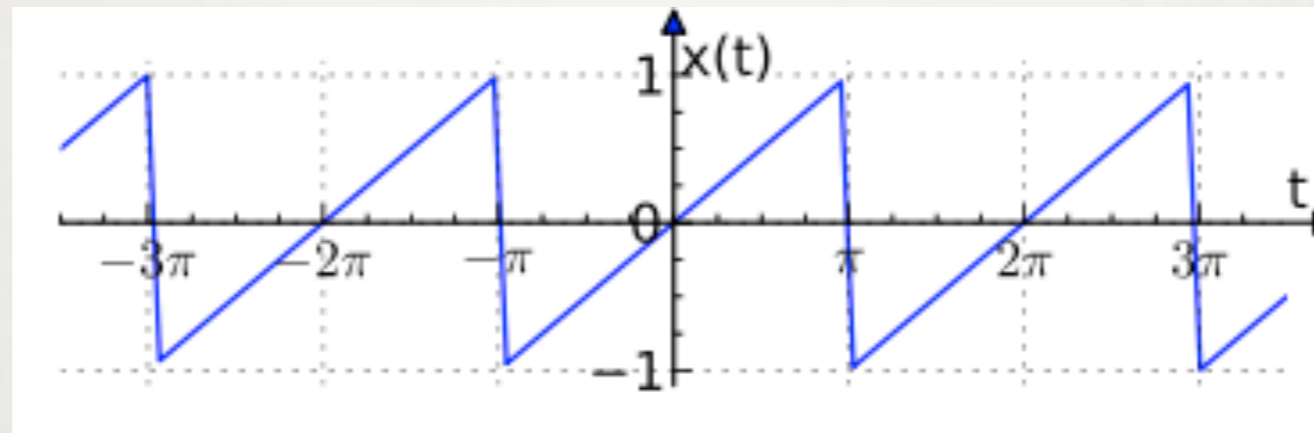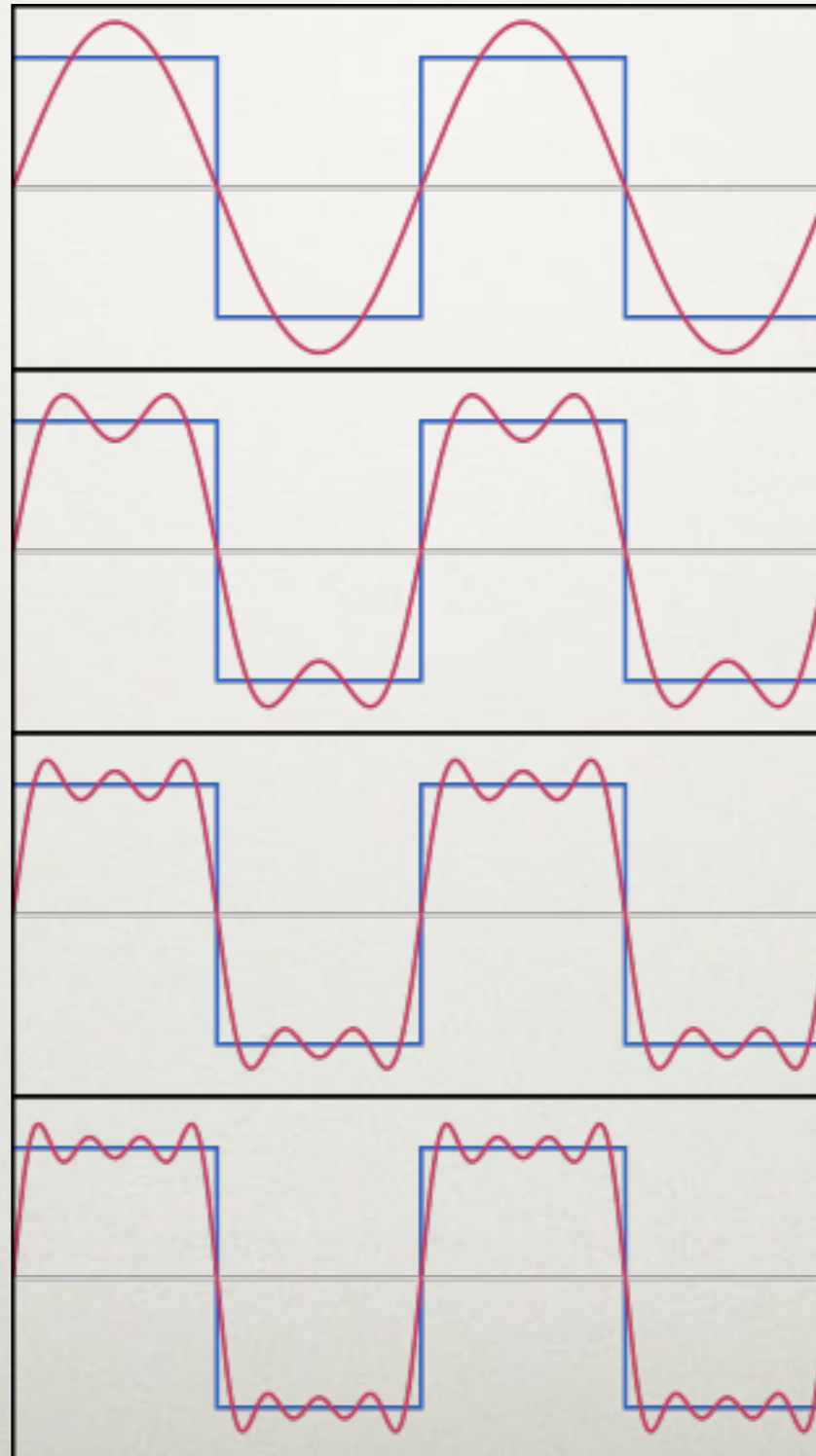
$$= 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx), \quad \text{for} \quad x - \pi \notin 2\pi\mathbb{Z}.$$

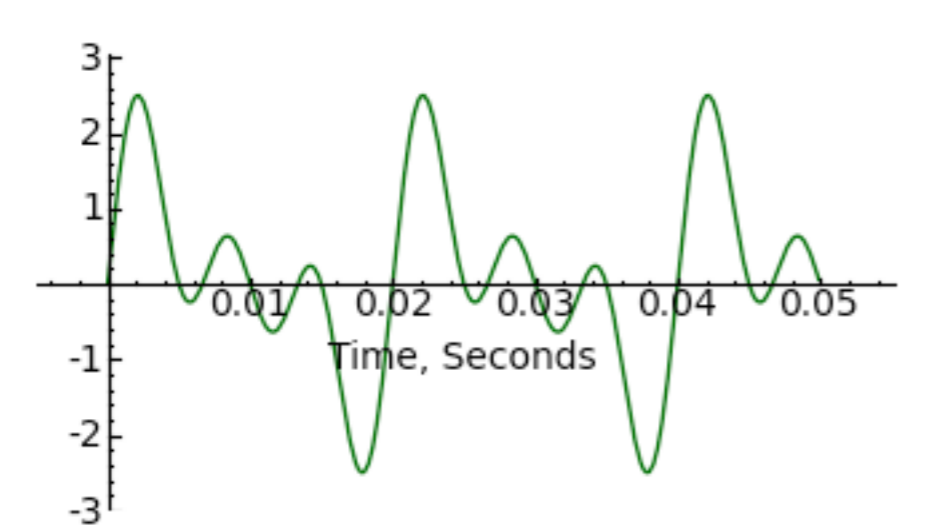# The first four Fourier series approximations for a square wave.

Fourier Transform

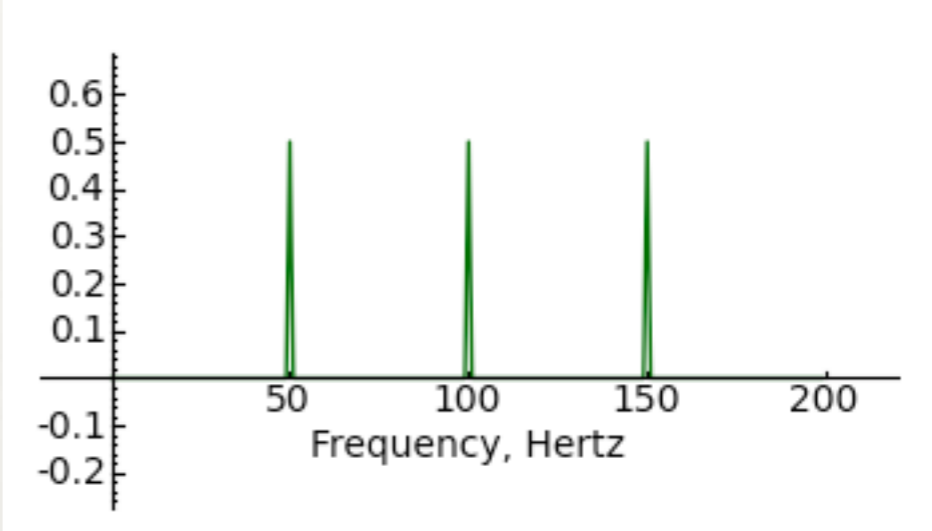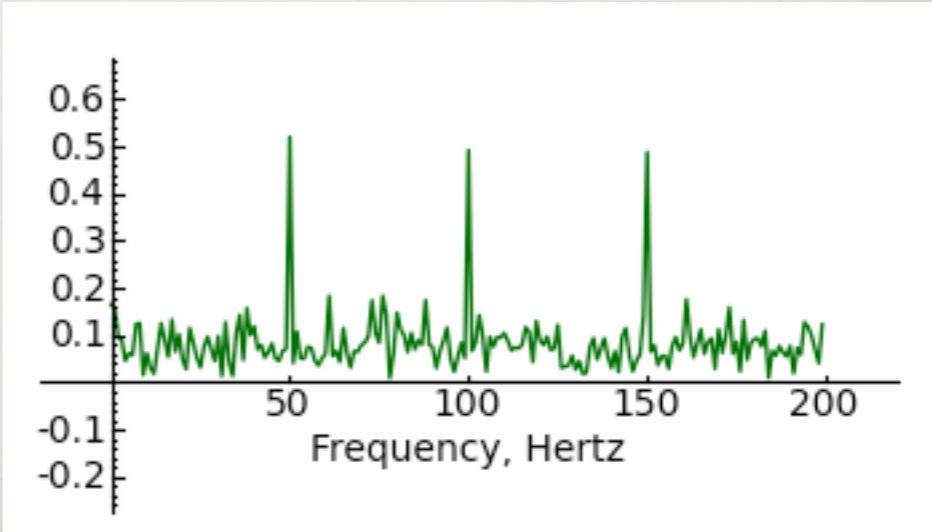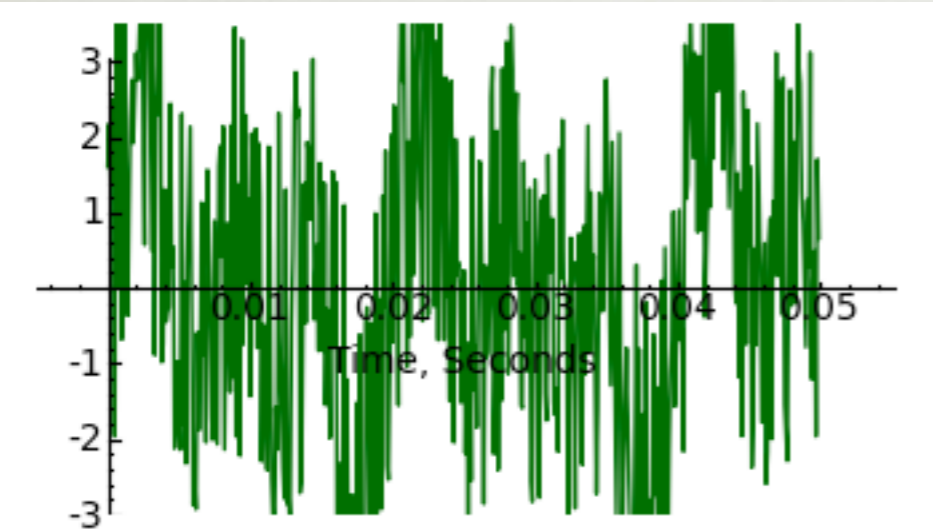$$f(y) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x y} dx \qquad f(x) = \int_{-\infty}^{\infty} f(y)\, e^{2\pi i x y} dy \qquad e^{2\pi i \theta} = \cos 2\pi\theta + i\sin 2\pi\theta$$

Time Domain

Frequency Domain

Interference (간섭)

Interference (간섭)

Constructive & Destructive interference
©2008 HowStuffWorks

Amplification

In phase wavelength

Cancellation

180° out of phase

Node   Antinode

Two-Point Source Interference Pattern

● = Maximum Pressure
● = Minimum Pressure

Source 1   Source 2

Sources
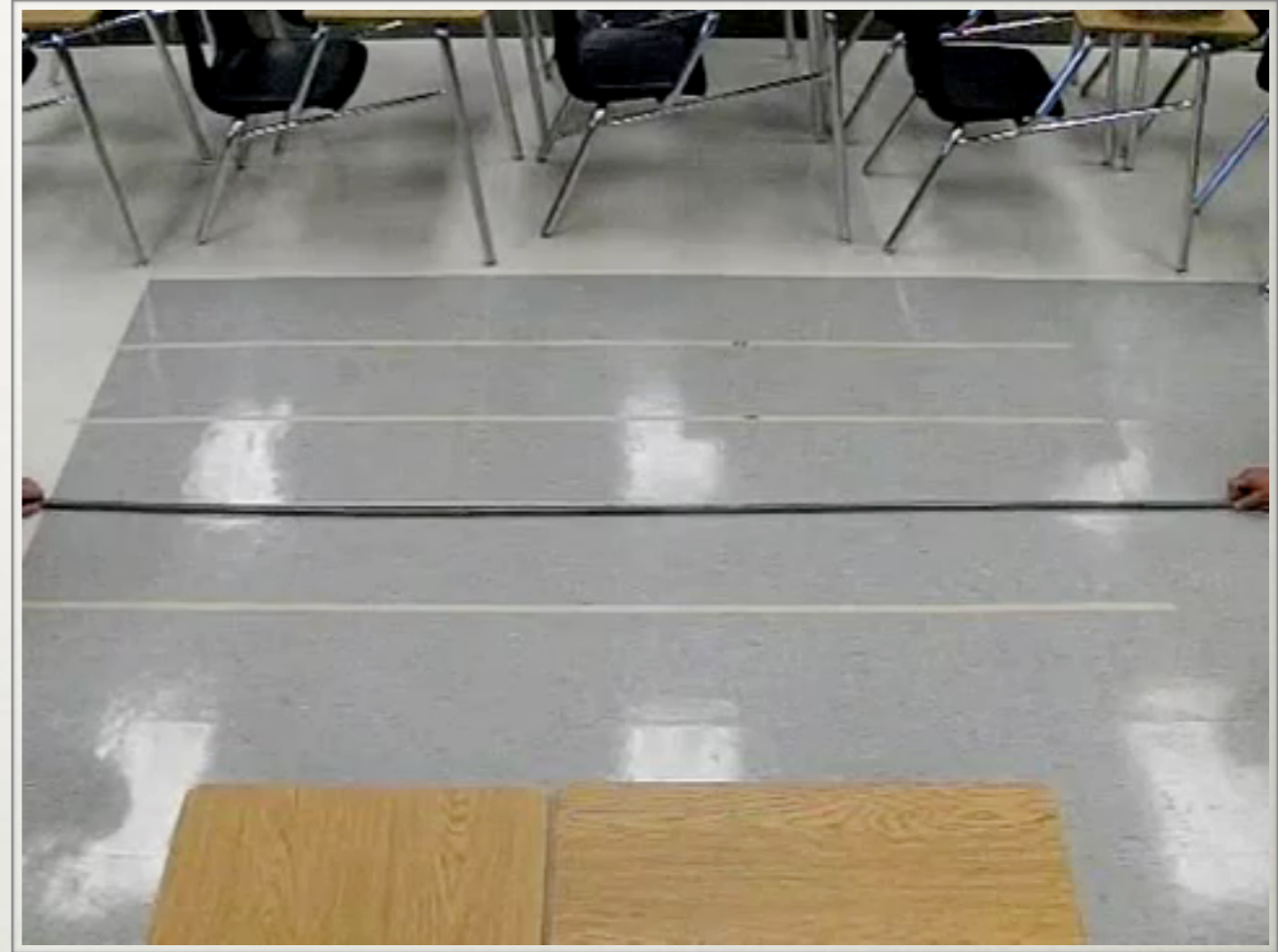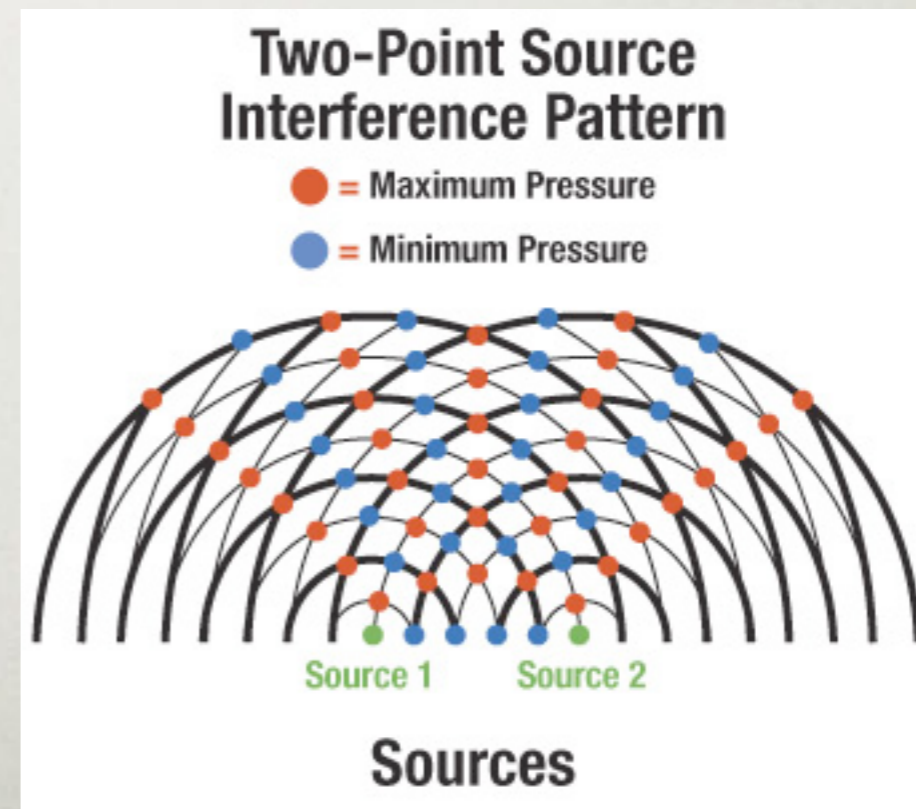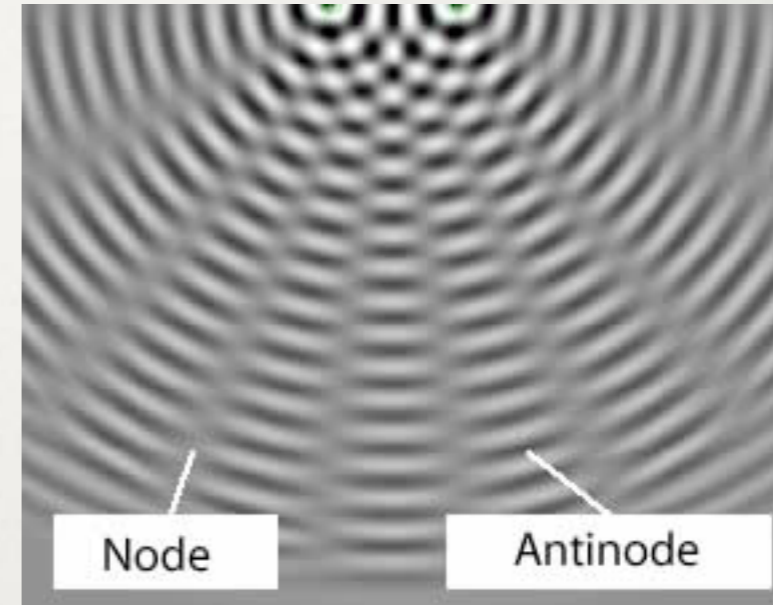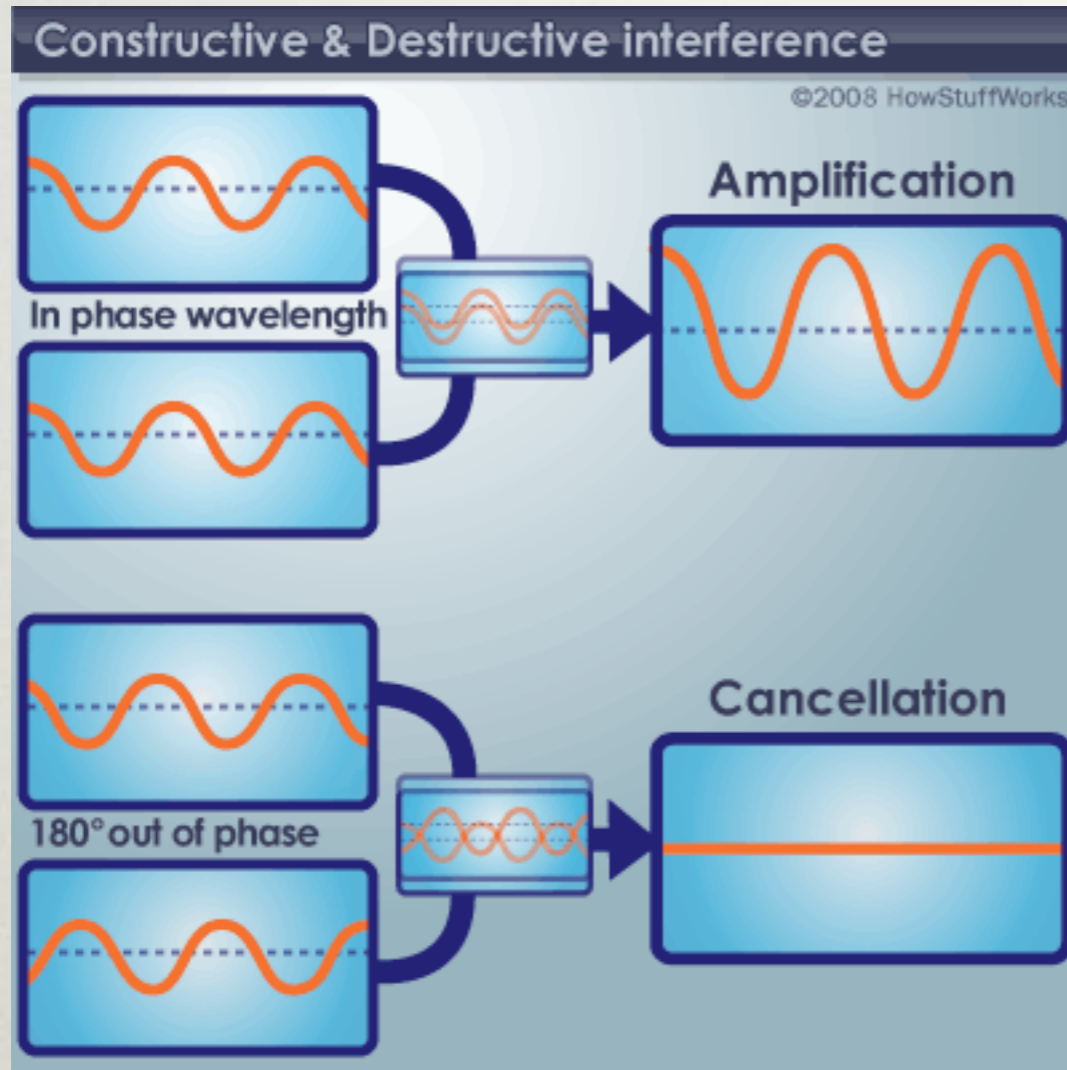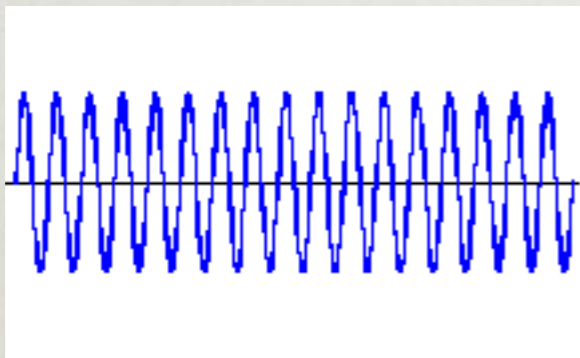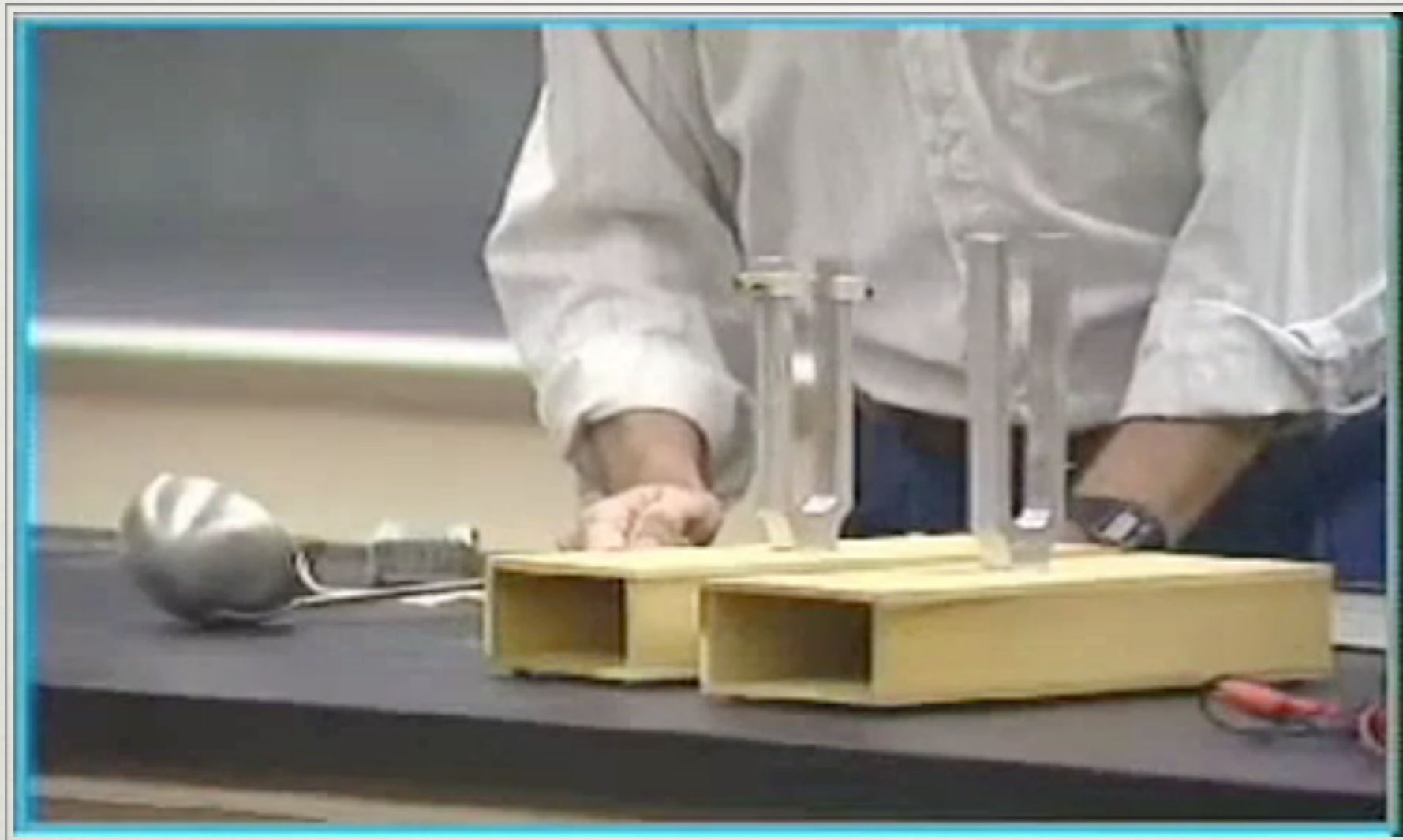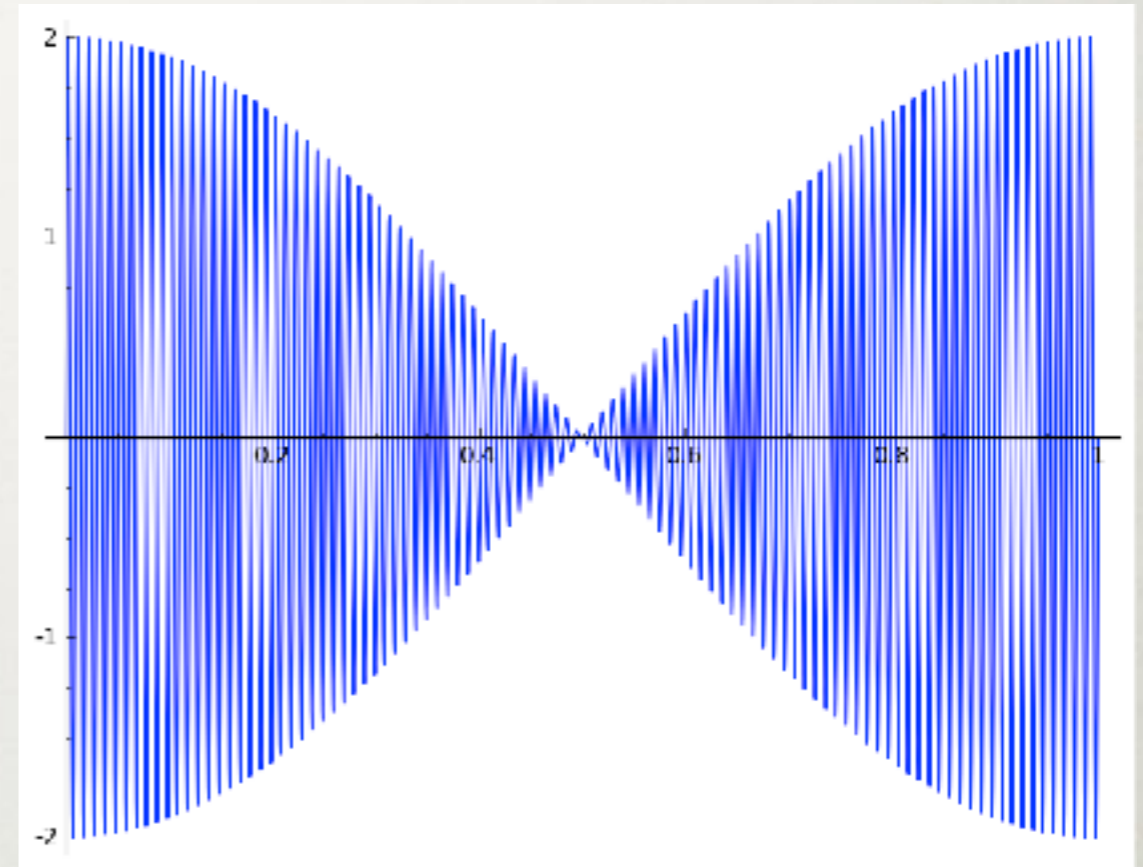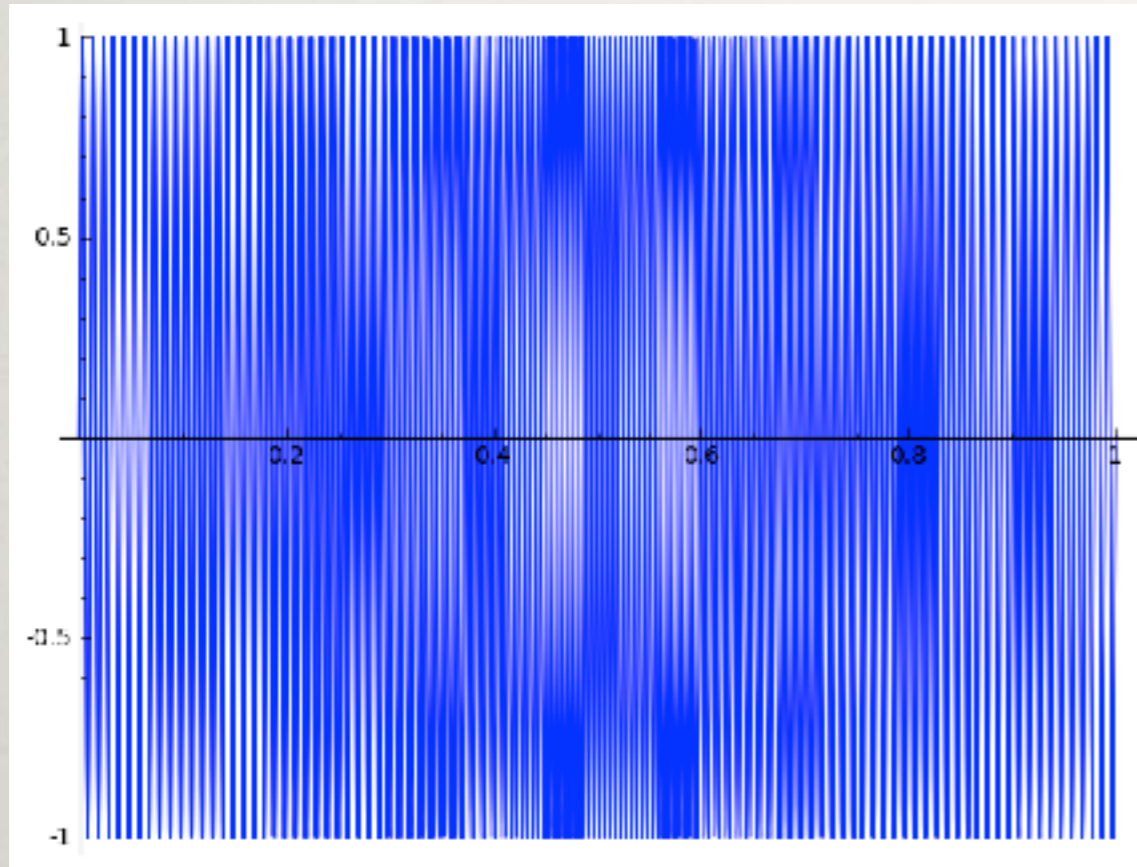
조율 (Tuning)

- **각 음간의 간격은 일정한 비율로 정해짐:**

  - Two notes separated by a perfect fifth have a frequency ratio of 3:2.

  - Notice that 2nd and 3rd harmonic on string are perfect 5th

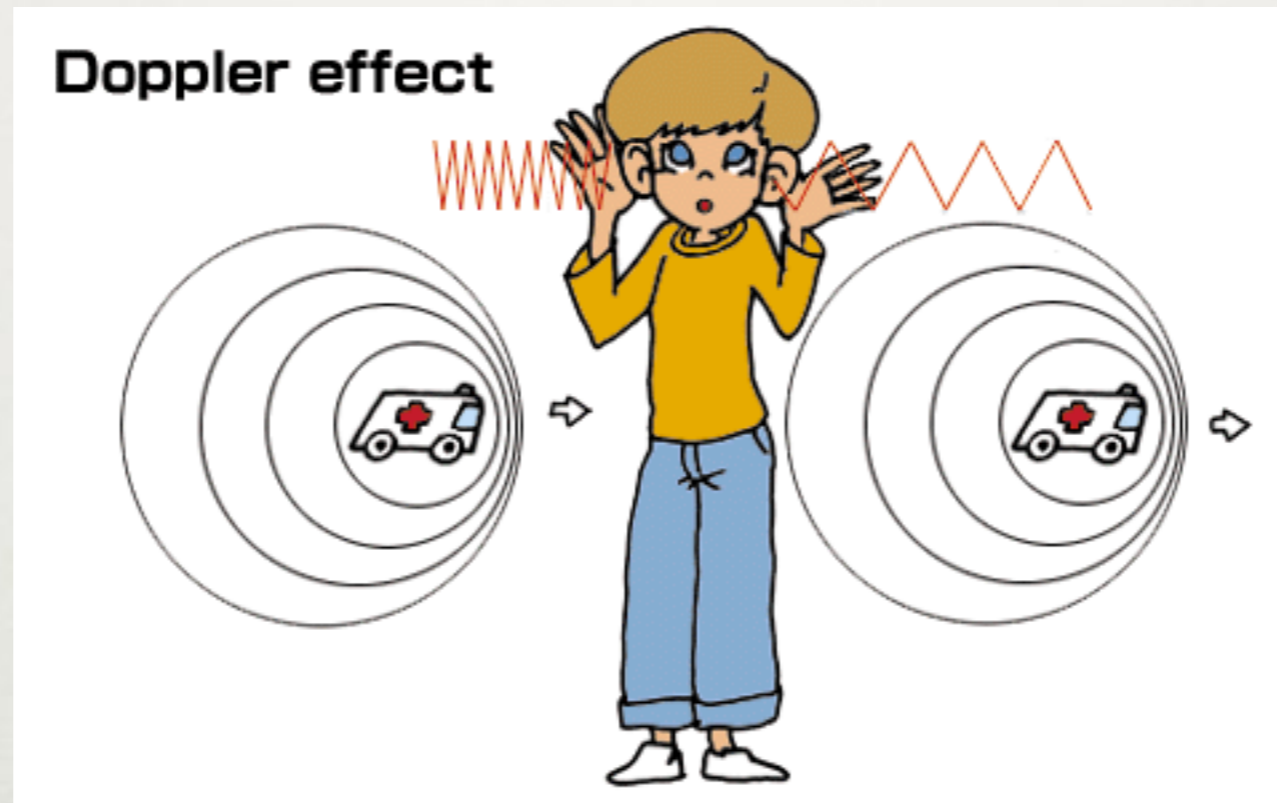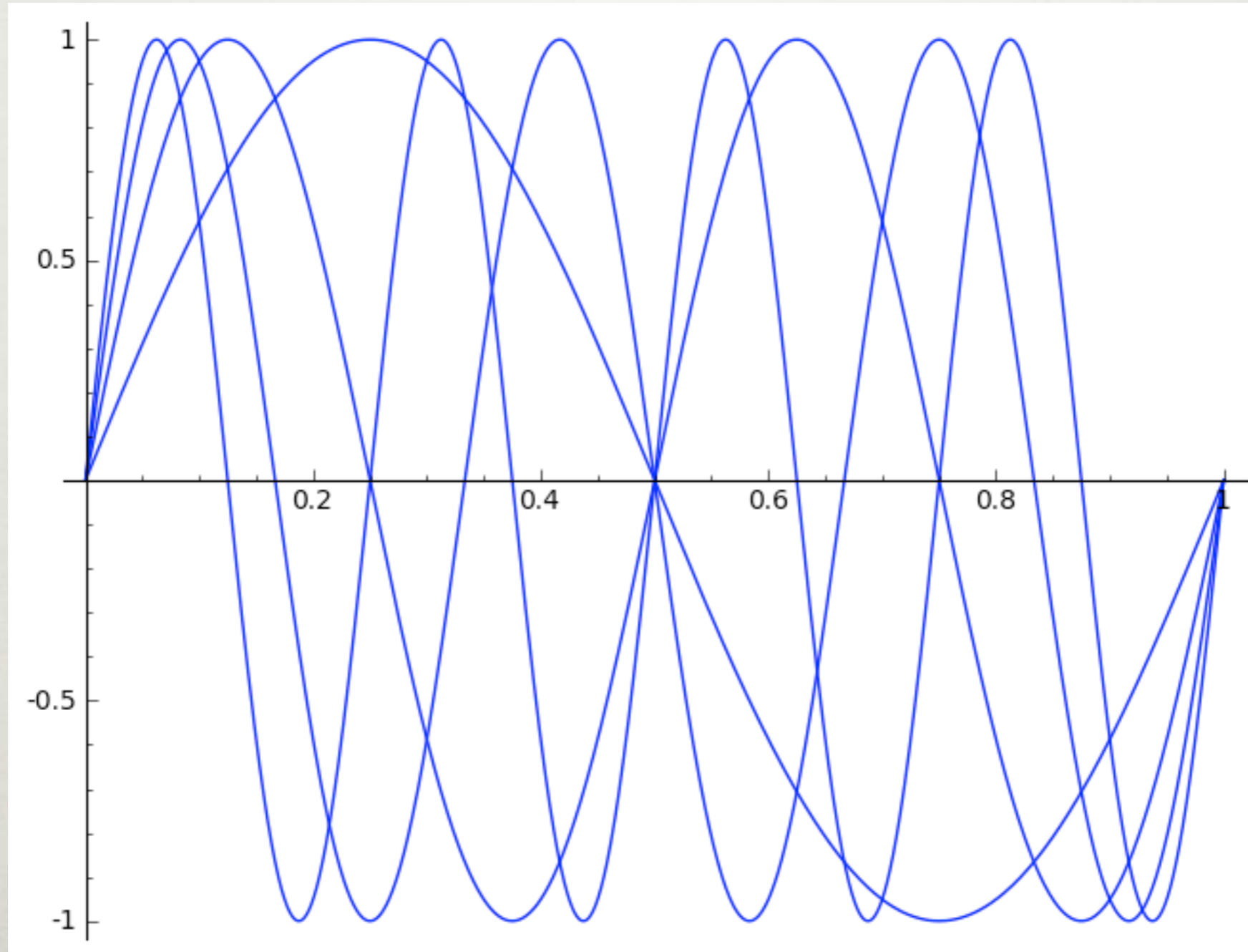- **맥놀이 (beating)**
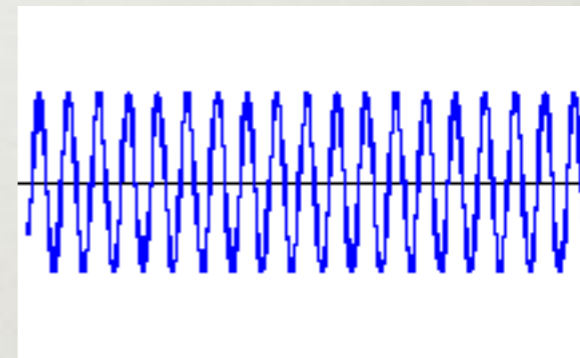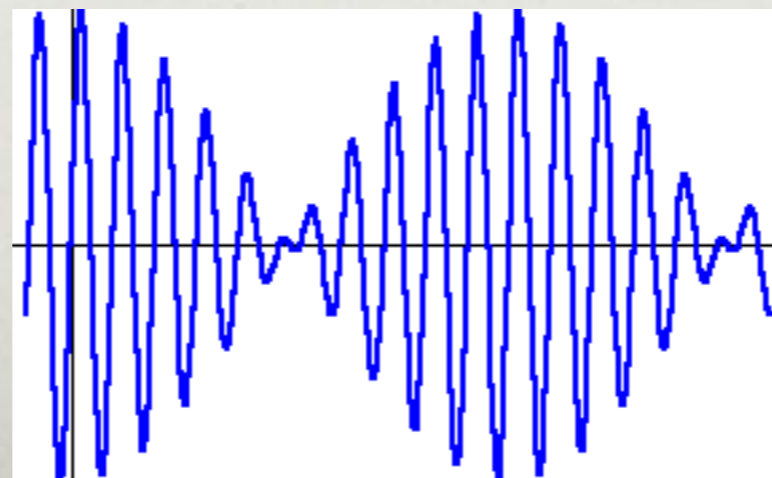
Beating: an interference in time

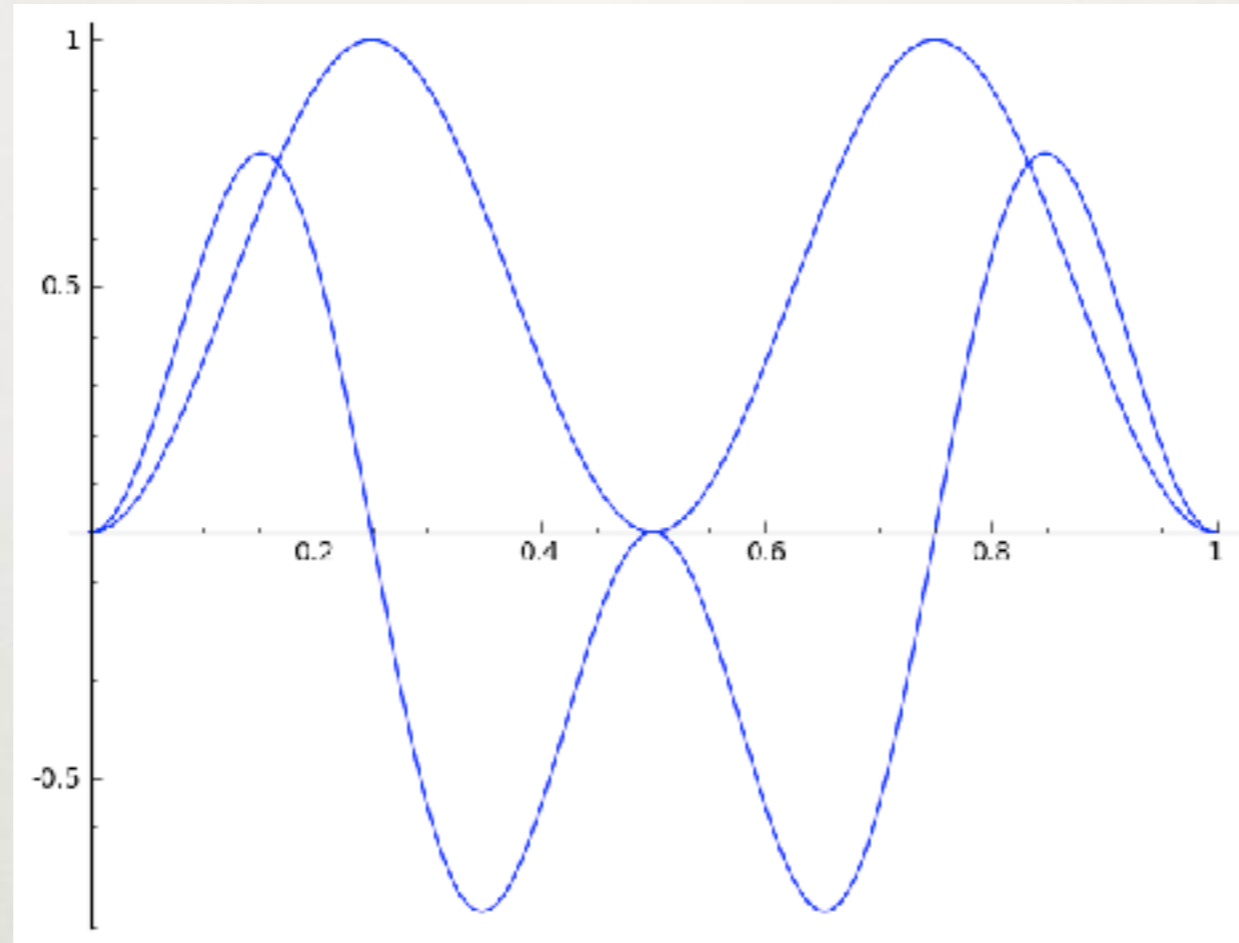# $\sin(2\pi*99t)+\sin(2\pi*100t)$

Doppler Effect

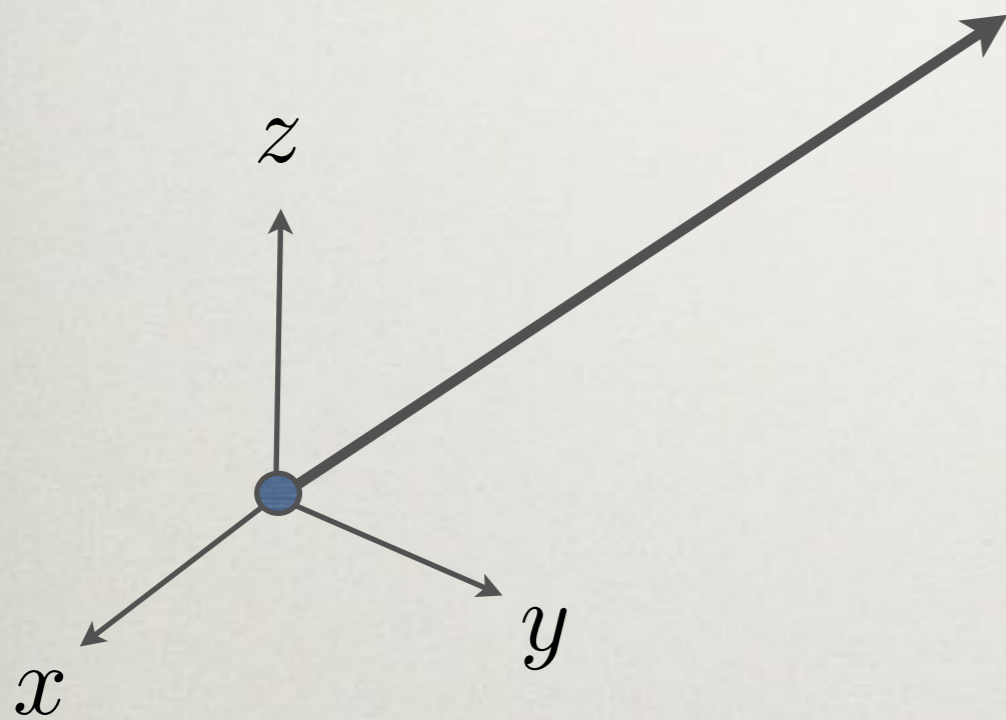# Fourier Series and vectors

norm and orthogonality

$$\int_0^{2\pi} \sin mt \cdot \sin nt = \pi \delta_{mn} \qquad (m, n \neq 0)$$

Vector space

$$\mathbf{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\hat{x} \cdot \hat{x} = 1$$

$$\hat{x} \cdot \hat{y} = 0$$

$$\mathbf{r} = \sum_{i=1,2,3} x_i \hat{e}_i$$

z

y

x

vector space

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \sin nt$$

$$f(x) = (a_0, a_1, a_2, \dots)$$

$$f(x) = \sum_{n=0}^{\infty} a_n \hat{e}_n$$